

transmission coefficients for Upsala, Sweden (18), latitude  $59^{\circ} 51' N.$  are, in general, only about 0.005 less than the transmissions I have computed for latitude  $60^{\circ} N.$

Average daily totals of the radiation received on a horizontal surface are available for Stockholm, Sweden, latitude  $59^{\circ} 21' N.$ , and for Habana, Cuba, latitude  $23^{\circ} 09' N$  (18), both of which have a semimarine climate. The records for Stockholm cover a single year, those for Habana about 15 months. The Stockholm records give daily totals only a few per cent less than the computed values of Table 4 for latitude  $60^{\circ} N.$ , except for May and June. The published curve of daily totals [(18) Figure 2], shows a decided depression for these two months. The daily totals for Habana [(18) Figure 1], are less than the computed values of Table 4 for latitude  $20^{\circ} N.$ , for the months October to February, inclusive, and more than the computed values for March to September, inclusive. We would expect the total radiation at Habana, latitude  $23^{\circ} 09' N.$ , to be more than at latitude  $20^{\circ} N.$  in summer and less in winter.

It seems evident that with reliable climatological data the radiation intensity over the oceans may be computed with considerable accuracy; but the values here given must not be accepted as final.

## REFERENCES

- (1) THOMSON, ANDREW.  
1927. Solar radiation observations at Apia, Samoa. Monthly Weather Review, 55 : 226.
- (2) LINKE, F.  
1924. Results of measurements of solar radiation and atmospheric turbidity over the Atlantic Ocean and in Argentina. Monthly Weather Review, 52 : 157.
- (3) GORCZYŃSKI, LADISLAUS.  
1923. Report of the Polish actinometric expedition to Siam and the equatorial regions. Bulletin Météorologique, Institut Central Météorologique à Varsovie, Nr. 9-10, p. 90. (English summary.)
- (4) WESTMAN, J.  
1904. Mesures de l'intensité de la radiation solaire faites en 1899 et en 1900 à la station d'hivernage suédoise à la baie de Treurenberg, Spitzbergen. Mission scientifique pour la mesure d'un arc de méridien au Spitzberg entreprises en 1899-1902 sous les auspices des gouvernements Suédois et Russe. Mission suédoise. Tome 2, Section VIII, B. Stockholm.
- (5) INTERNATIONAL POLAR EXPEDITION, 1882-1883 (France).  
1885. Observations des radiations solaires. Mission scientifique du Cap Horn, 1882-1883. Tome 2. Meteorologie, Secunde partie, s. 59-73, Paris.
- (6) ABBOT, C. G. and others.  
1913. Annals Astrophysical Observatory, III, 141-143. Washington.
- (7) KING, LOUIS VESSAT.  
1913. On the scattering and absorption of light in gaseous media with applications to the intensity of sky radiation. Phil. Trans. R. Soc. London. A. 212, 375, London.
- (8) LORD RAYLEIGH.  
1899. On the transmission of light through an atmosphere containing small particles in suspension, and on the origin of the blue of the sky. Phil. Mag. 47, 375. London.
- (9) ABBOT, C. G. and others.  
1923. The distribution of energy in the spectrum of the sun and stars. Smith. Misc. Coll. V. 74, No. 7, Washington.
- (10) ABBOT, C. G.  
1927. Smithsonian solar radiation researches. Sonderdruck aus Gerland's Beiträge zur Geophysik. Bd. 16, Heft 4, Leipzig.
- (11) FOWLE, F. E., Jr.  
1917. Water vapor transparency to low temperature radiation. Smithsonian Misc. Coll. Vol. 68, No. 8, Washington.
- (12) FOWLE, F. E., Jr.  
1915. The transparency of aqueous vapor. Astrophysical Journal, 42, 394.

- (13) LINKE, FRANZ, and BODA, KARL.  
1922. Vorschläge zur Berechnung des Trübungsgrades der Atmosphäre aus den Messungen der Intensität der Sonnenstrahlung. Met. Zeit, 39, 161.
- (14) REED, W. W.  
1926. Climatological data for the West Indies. Monthly Weather Review, 54, 133, Washington. 1927. Climatological data for the tropical islands of the Pacific Ocean (Oceania), Monthly Weather Review Supplement No. 28, Washington.
- (15) KIMBALL, H. H.  
1925. Records of total solar radiation and their relation to daylight intensity. Monthly Weather Review, 52 : 475, Table 1.
- (16) ÅNGSTRÖM, A.  
1922. Note on the relation between time of sunshine and cloudiness in Stockholm, 1908-1920. Arkiv för matematik, Astronomi och Fysik, Band 17, No. 15, Stockholm.
- (17) KIMBALL, HERBERT H.  
1919. Variations in the total and luminous solar radiation with geographical position in the United States. Monthly Weather Review, 47 : 780. (Figure 9.)
- (18) KIMBALL, HERBERT H.  
1927. Measurements of solar radiation intensity and determinations of its depletion by the atmosphere, with bibliography of pyrheliometric observations. Monthly Weather Review, 55 : 155-169. Washington.

HEATING AND COOLING OF WATER SURFACES<sup>1</sup>

By GEORGE F. McEWEN

(Abstract)

Under this title a brief report was read based upon a 50-page manuscript entitled: "Mathematical Theory of Vertical Temperature Distribution in Water under the Action of Radiation, Evaporation, and the Resulting Convection or Mixing." (Derivation of a general theory, and its illustration by means of numerical applications to reservoirs, lakes, and oceans.)

Since this manuscript will be ready in the summer of 1928 for publication in full as a bulletin (technical) of the Scripps Institution of Oceanography, La Jolla, Calif., by the University of California Press, only a brief abstract is presented here.

A mechanism has been devised involving the sinking of surface masses of water rendered relatively heavy, the evaporation, conduction through the air, and back radiation, and a compensating ascent of lighter, warmer masses. The mathematical theory of this mechanism of the downward diffusion of surface cooling led to a pair of simultaneous differential equations involving turbulence, rate of surface cooling, rate at which solar radiation penetrates the surface, rate of vertical distribution of temperature and salinity, and rate of vertical flow of the water.

Methods have been worked out for applying these equations to numerical data, without the need of their general solution, which has not been attempted. Three integrals appearing in these equations and depending upon the vertical variation of specific gravity have been tabulated to facilitate numerical applications.

Numerical results have been obtained by applying the theory to serial temperatures in a tank of water, to serial temperatures of Lake Mendota, Wis., and to serial temperatures and salinities in the Pacific Ocean near San Diego, Calif. From such observations the theory provided a means of estimating the rate of penetration of solar radiation through the water surface, the rate of surface cooling, and, therefore, an approximate estimate of evaporation and the rate of vertical flow of the water.

<sup>1</sup> The full title as given by the author is "The rate at which solar radiation penetrates the surface of lakes and oceans, and the rate at which the surface loses heat as deduced from serial temperature-observations."—Ed.

The upwelling velocity in the Pacific near San Diego was thus estimated to be about 25 meters per month during the summer, and the rate of penetration of solar radiation thus found was in good agreement with results obtained by independent methods.

By applying the same mechanism of downward diffusion to the distribution of salinity, and combining the resulting equations with those pertaining to temperature, the surface cooling due to evaporation and other causes can be estimated separately. This means of determining the rate of evaporation from the ocean by means of serial observations of temperature and salinity between the surface and the hundred-meter level, while

theoretically possible, has not yet been applied. An approximate estimate of ocean evaporation from the rate of surface cooling can be made by supplementing the serial observation with observations on an evaporating pan containing sea water.

Numerical or graphical integration of the equations after the various physical magnitudes have been found as indicated above should reproduce the subsequent changes in temperatures and salinities from their initial values. If the upwelling velocity is not included, integration should yield "normal" values; that is, values of temperature and salinity to be expected in the absence of a general flow of the water.

## A NEW ANALYSIS OF THE SUN SPOT NUMBERS

By DINSMORE ALTER

[University of Kansas, Lawrence, Kans., November 16, 1928]

During the past 10 years the writer has worked a great deal with analyses of data and has spent much time on the sun spot numbers. However, except for one brief paper on the 11-year means (1a), read before the Astronomical Society in 1921, he obtained no results worthy of publication by any of the older methods.

With the development of the equations used in the correlation periodogram (1b) last year it seemed worth while once more to try the problem. All previous analyses had depended on repetitions of a sine curve of assumed period, the best of these being those made by the Schuster method. For such a method the number of data was far too small, either to prove the existence or nonexistence of fairly constant periodicities (1c). The new method, however, using not only such a curve but all its harmonics simultaneously, promised to require less data for a decision. Accordingly a thorough analysis was made. Since the complete method was published in this journal it will not be developed here.

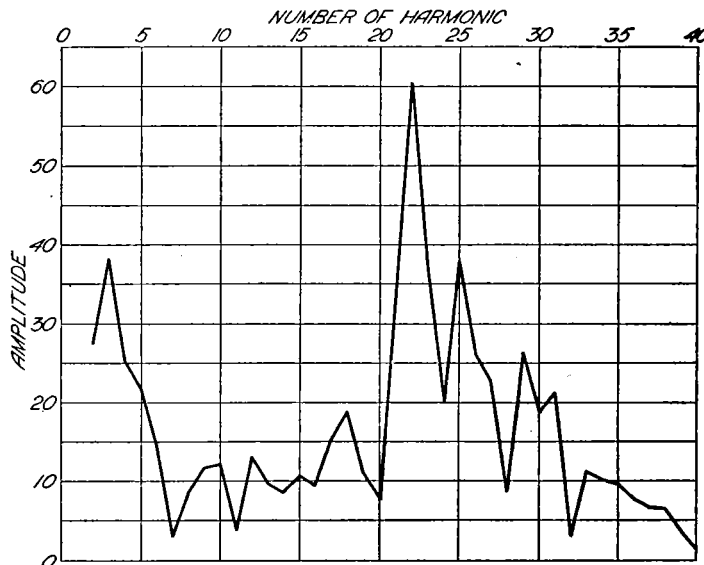
The unsmoothed Wolf-Wolfer numbers for the years 1749-1926 were formed into 6-month means, giving 356 data. The periodogram was computed using logs, varying by 6-month steps, from  $12\frac{1}{2}$  to  $142\frac{1}{2}$  years for the separate correlations. The number of pairs of data used therefore, in computing each correlation coefficient ranged from 331 down to 71. Beyond the latter number but little accuracy would have been secured. The periodogram is reproduced as Figure 1.

Naturally the first feature of the periodogram to strike the eye is the series of crests at a little more than 11-year intervals. Examination will show, however, that their intervals do not even approximate the generally accepted value of  $11\frac{1}{8}$  years but average 11.37 years and are quite consistent in grouping around this figure. In other words, although the number of maxima and minima usually considered by investigators to be principal ones is such as to give approximately an  $11\frac{1}{8}$ -year mean, the shape of the curve is such that the best correlations occur after intervals of more than  $11\frac{1}{8}$  years. This checks with the long value assumed by Mount Wilson.

The next significant feature of the periodogram is the variation in amplitude of its swing between minimum and maximum. It reaches a minimum amplitude at about 33 years, a maximum at about 65 or 70, and a very pronounced minimum at about 126 years. The curve as a whole also swings about the zero line from a minimum at approximately 40 years to a maximum at about 85 and another minimum near 126. The latter feature is evidently due to a cycle (either accidental or significant) of about 85 years.

The variation in amplitude is far too great to be accidental. Perhaps we have no exact periodicity, but a tendency toward lengthening and shortening of the cycle such tendencies persisting through considerable number of years. However, the changes resemble so closely the familiar best pattern made by superimposed periodicities that it is worthwhile investigating the hypothesis that they actually are such.

The amplitude has decreased nearly to zero at 126 years. Such a pattern could come only through the superposition of periodicities, which are harmonics of



No. 1. Correlation periodogram of sun spot numbers

twice this period. If only such a primary and a 126-year period existed, the pattern would have shown a steady decrease to zero at this point instead of the maximum at 85 years. It is obviously easy to find three harmonics of 252 years which would give the secondary and primary minima and the maximum observed. This was done early in the investigation, but the more logical plan is to compute all the harmonics of 252 years and find the amplitude of each. *One thing which the pattern definitely tells us is that, if the sun spot variation is the result of superposition of fairly constant periodicities, any large ones of length less than a century must be harmonics of approximately 252 years.*

These harmonics of 252 years were, therefore, all computed beginning with the second of period 126 years and